Decidability

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Binary Strings for TM's

- We shall restrict ourselves to TM's with input alphabet $\{0,1\}$.
- Assign positive integers to the three classes of elements involved in moves:
  1. States: q_1 (start state), q_2 (final state), q_3, ...
  2. Symbols X_1 (0), X_2 (1), X_3 (blank), X_4, ...
  3. Directions D_1 (L) and D_2 (R).

Binary Strings for TM's

- Suppose $\delta(q_i, X_j) = (q_k, X_l, D_m)$.
- Represent this rule bu string $0^i 1 0^j 1 0^k 1 0^l 1 0^m$.
- **Key point:** Since integers i, j, ... are all > 0, there cannot be two consecutive 1's in these strings.

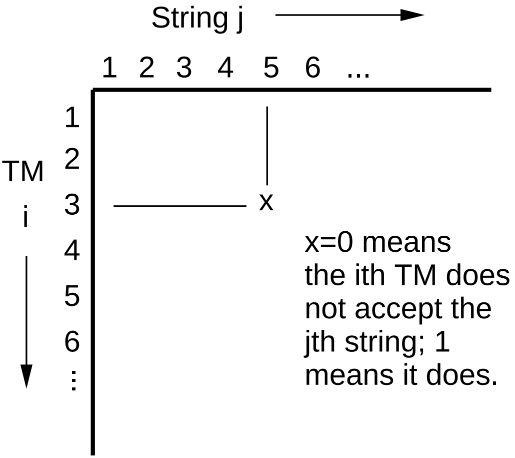
Binary Strings for TM's

- Represent a TM by concatenating the codes for each of its moves, separated by 11 as punctuation.
  - That is: $\text{Code}_1 11 \text{Code}_2 11 \text{Code}_3 11 \ldots$

Enumerating TM's and Binary Strings

- We can uniquely encode binary strings as integers.
- Thus, it makes sense to talk about the ith binary string and about the ith Turing machine.
- **Note:** If i makes no sense as a TM, assume the ith TM accepts nothing.

Table of Acceptance



Diagonalization Again

- whenever we have a table like the one on the previous slide, we can diagonalize it.
  - That is, construct a sequence D by complementing each bit along the major diagonal.
- Formally, $D = a_1 a_2 \ldots$, where $a_i = 0$ if the (i,i) table entry is 1, and vice-versa.

The Diagonalization Argument

- Could D be a row (representing the language accepted by a TM) of the table?
- Suppose it were the jth row.
- But D disagrees with the jth row at the jth column.
- Thus D is not a row.

Diagonalization

- Consider the diagonalization langauge $L_d = \{w \mid w \text{ is the } i\text{th string, and the } i\text{th TM does not accept } w\}$.
- We have shown that L_d is not a recursively enumerable language, i.e., it has no TM.