CS154: Midterm Exam

Total points: 80 (choose any 8, extra credit if you answer more)

Problem 1. These look like true/false questions, but they are really short answer questions. Decide if the following statements are TRUE or FALSE and give short reasons for your choice. (10 points)

a) $\{0^n 1^n \mid n \geq 0\} \cap R$, where R is a regular language, is never regular.

b) Suppose $\Sigma = \{0, 1\}$, and let $\mathsf{sort}(x)$ be the function that reorders the symbols in x in numerical order. Let $\mathsf{sort}(L) = \{\mathsf{sort}(x) \mid x \in L\}$. For example, if L={0,1,01,10,101,0110}, then $\mathsf{sort}(L)$={0,1,01,011,0011}. Regular languages are closed under sort.

Problem 2. These look like true/false questions, but they are really short answer questions. Decide if the following statements are TRUE or FALSE and give short reasons for your choice. (10 points)

a) If L is a **finite** context-free language, then $\bar{L}$ (the complement of L) must be context-free.

b) If every state of an NFA N is accepting, then $L(N) = \Sigma^*$.

Problem 3. Design a DFA that accepts strings over {0,1} containing 101 as a substring. (10 points)

Note: Draw the graph. **Do not** give the transition table.

Problem 4. Prove that there exists an integer whose decimal representation consists entirely of 1's, and which is divisible by 1987. (10 points)

Hint: If p is a prime such that $p|(xy)$ and p does not divide y, then $p|x$. You should only consider the remainders modulo 1987 of integers whose decimal representation consists entirely of 1's and use the pigeonhole principle.

Problem 5. Show that regular languages are **not closed** under infinite union. (10 points)

Note: Infinite union means a union of an infinite family of sets. For this problem, you need to come up with an infinite family of regular sets whose union is *not* regular.
Hint: Start with a non-regular language and break it down into an infinite family of regular sets.

Problem 6. Design a PDA for strings over {0,1} of the form $0^n 1^{2n}$, where $n \geq 0$. (10 points)

Note: You can either give a PDA that accepts by *final state* or by *empty stack*.

Problem 7. Design a PDA for the set of all palindromes over {0,1}. (10 points)

Note: You can either give a PDA that accepts by *final state* or by *empty stack*.
Hint: You should first write out the corresponding CFG and then convert it to a PDA.

Problem 8. Prove that $2^{243} + 1$ is divisible by 729. (10 points)

Hint: Try to prove the more general statement that $2^{3^n} + 1$ is divisible by 3^{n+1}. Recall that $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^{ij} = (a^i)^j$.

Problem 9. Minimize the DFA shown in Figure 1 by marking distinguishable states in a table and then draw the minimized DFA. (10 points)

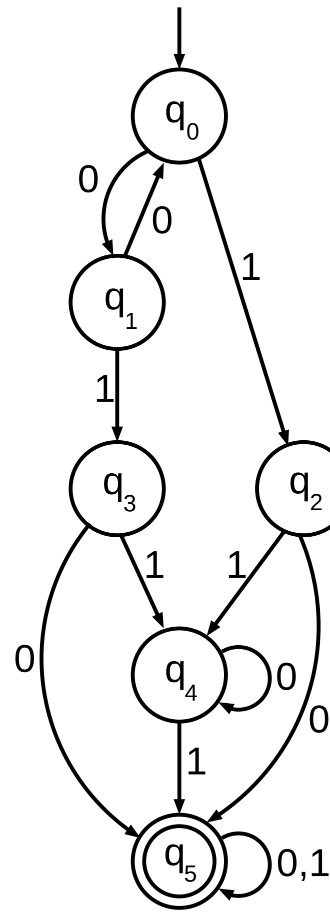

Figure 1: DFA for problem 7.

Problem 10. Prove that the set of all strings over {0,1} of the form $w\overline{w}$, where $\overline{w}$ is formed from w by replacing all 0's by 1's, and vice-versa is not regular. For example, $\overline{011} = 100$ and 011100 is an example of a string in the language. (10 points)