CS154: Homework #2

Due: Wednesday, July 18, 2012 by 5PM

Problem 1. For any DFA, we extend the transition function δ by breaking the input string $\mathbf{w} = \mathbf{xa}$ during the inductive step, where $\mathbf{x}$ is any string followed by a single symbol $\mathbf{a}$. However, we informally think of δ as describing what happens along a path with a certain string of symbols, and if so, then it should not matter how we break the input string. Show that in fact,

$$\delta(q, \mathbf{xy}) = \delta(\delta(q, \mathbf{x}), \mathbf{y})$$

for any state q and strings $\mathbf{x}$ and $\mathbf{y}$.

Problem 2. Give DFA's accepting the following languages over the alphabet $\{0, 1\}$.

a) The set of all strings such that each block of **four consecutive** symbols contains **at least two** 0's.

b) The set of strings such that the number of 0's is divisible by 3, **and** the number of 1's is divisible by 3.

Problem 3. Consider the following ε-NFA.

	ε	a	b	c
$\rightarrow p$	$\{q,r\}$	$\emptyset$	$\{q\}$	$\{r\}$
q	$\emptyset$	$\{p\}$	$\{r\}$	$\{p,q\}$
r^*	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Recall that starred states denote accept/final states.

a) Compute the ε-closure of each state.

b) Give all the strings of length three or less accepted by the automaton.

c) Convert the automaton to a DFA.

Problem 4. Write regular expressions for the following languages. In all parts the alphabet is $\{0, 1\}$.

a) $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$.

b) $\{w \mid w$ has at most one pair of consecutive 1's$\}$.

c) $\{w \mid$ the number of 0's in w is divisible by 3$\}$.

d) $\{w \mid$ every pair of adjacent 0's in w appears before any pair of adjacent 1's$\}$.

e) $\{w \mid$ every odd position of w is a 1$\}$.

Problem 5. Prove that the language $\mathbf{L} = \{w \mid w \in \{0, 1\}^*$ is not a palindrome$\}$ is not regular.