CS154: Homework #1

Due: Wednesday, July 11, 2012 by 5PM

Problem 1. *Given 8 distinct natural numbers, none greater than 15, show that at least three pairs of them have the same positive difference (the pairs need not be disjoint as sets).*

- **Note:** Recall that the set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$.

Problem 2. *Prove that the number $111\ldots 11$ (243 ones) is divisible by 243.*

- **Hint:** Try proving that a number written with 3^n ones is divisible by 3^n, for all $n \geq 1$.

Problem 3. *Draw a DFA to show that the language L over the alphabet $\{0, 1\}$ consisting of runs of even numbers of 0's and odd numbers of 1's is regular. Also draw the transition table for this DFA.*

- **Note:** A *run* of an alphabet is a continuous sequence of that alphabet, for example, here are few strings with runs of even numbers of 0's: $10011, 11100001, 11001000011$. You are required to give a DFA that accepts strings containing runs with even numbers of 0's **and** runs with odd numbers of 1's.

Problem 4. *Let $L \subset \{0, 1\}^*$ be the language of all strings such that there are two 0's separated by a number of positions that is a **non-zero** multiple of 5. Construct an NFA for this language.*

- **Note:** Each position between the two 0's contains an arbitrary symbol (either 0 or 1). For example, 1001110 is not in L, but $1\underline{0}111\underline{0}$ and $\underline{0}10101\underline{0}1$ are both in L.

Problem 5. *Consider an NFA $N_1 = (Q, \Sigma, \delta_N, q_0, F)$ with language $L_1 = L(N_1)$. Define a new NFA $N_2 = (Q, \Sigma, \delta_N, q_0, Q - F)$, i.e., going from N_1 to N_2 the final states become non-final, and vice-versa. Prove or disprove that the language $L_2 = L(N_2)$ is the **complement** of the language L_1, i.e., $L_2 = \Sigma^* - L_1$.*

- **Note:** If the statement is true, you must provide a formal proof that applies to all NFA's N_1. However, if the statement is false, then all you need to do is describe a specific NFA N_1 and show that the statement is incorrect when applied to this N_1.