CS154: Introduction to Automata and Complexity Theory

FINAL EXAMINATION — 12:15PM - 2:15PM, Saturday, 08/18/2012

Instructions: Answer **any 4 questions**. You have a total of **2 hours** and the maximum possible score is 80. The point allocation is roughly proportional to the time required to solve a problem, i.e., 1 point for each minute, with 20 minutes left over to reduce time pressure. We strongly recommend that you use a few minutes to go over the exam and plan your strategy. Please clearly indicate which four problems you are solving. You may receive extra credit if you solve more.

This exam is an **open book** and **open notes** exam, i.e., you are allowed to consult both the text-book, your class notes and homeworks, and any of the handouts from us. You are not permitted to refer to any other material (including, of course, online material). No use of computers, cell phones, etc. is permitted.

Problem 1. (20 points) Decide if the following statements are TRUE or FALSE. Also write **one line** justification after each answer.

a) For every language $L \in \mathbf{P}$, there is a polynomial-time reduction from L to the language $\{\varepsilon\}$ (i.e., problem instances are strings w, and the decision problem is: "*Is* $w = \varepsilon$*?*").

b) Suppose L_1 is *not* recursively enumerable and L_2 is finite. Then $L_1 \cap L_2$ is recursive.

c) The problem $L(G) \subseteq L(N)$ is decidable when G is a context-free grammar and N is a non-deterministic finite automaton.

d) There are context-free languages whose complements are not decidable.

e) If an algorithm is discovered that can solve some NP-complete problem on an NTM in a polynomial number of steps in the worst case, it means that $\mathbf{P} = \mathbf{NP}$.

Problem 2. (20 points) Let the tape alphabet be Γ. Consider the problem that given a Turing machine M, input string w and a symbol $X \in \Gamma$, when running on input w, will M ever write the symbol X on its tape. Show that this problem is undecidable. Is this problem recursively enumerable? Supply a proof of your answer.

Problem 3. (20 points) Assume $L_{\mathsf{e}} = \{M \mid L(M) = \emptyset\}$ is not recursively enumerable. Define another language L_{eq} as follows

$$L_{\mathsf{eq}} = \{< M_1, M_2 > \ \mid L(M_1) = L(M_2)\}$$

a) Prove that L_{eq} is not recursively enumerable.

b) Prove that $\overline{L_{\mathsf{eq}}}$ is not recursively enumerable.

Hint: For part b), try reducing from L_d.

Problem 4. (20 points) A *monotone clause* in a CNF formula is a clause consisting of all positive or negative literals. For example, $(x_1 + x_2 + x_3)$ and $(\text{-}x_1 + \text{-}x_2 + \text{-}x_3)$ are monotone clauses, but $(x_1 + \text{-}x_2 + \text{-}x_3)$ is not. The *clause-monotone SAT problem* is to decide the satisfiability of a CNF formula that consists only of monotone clauses. Prove that the clause-monotone SAT problem is NP-complete.

Hint: Reduce from 3-SAT. Split each non-monotone clause into positive and negative parts, and add new variables.

Problem 5. (20 points) A 2-coloring of a set S is a partition of S into two subsets S_1 and S_2 such that $S_1 \cap S_2 = \emptyset$ and $S_1 \cup S_2 = S$, i.e., in a 2-coloring each of the elements in S is assigned exactly one of the two colors. Let SET SPLITTING be the following decision problem:

Input: A set S, and a collection of subsets $T_1, T_2, \ldots, T_k$, where $T_j \subseteq S$ for $j = 1, \ldots, k$.

Question: Is there a 2-coloring of S, $S = S_1 \cup S_2$, such that for all $j = 1, \ldots, k$, $T_j \cap S_1 \neq \emptyset$ and $T_j \cap S_2 \neq \emptyset$? Prove that SET SPLITTING is NP-complete.

Hint: Try reducing from 3-SAT. Use the literals as elements in the set S.