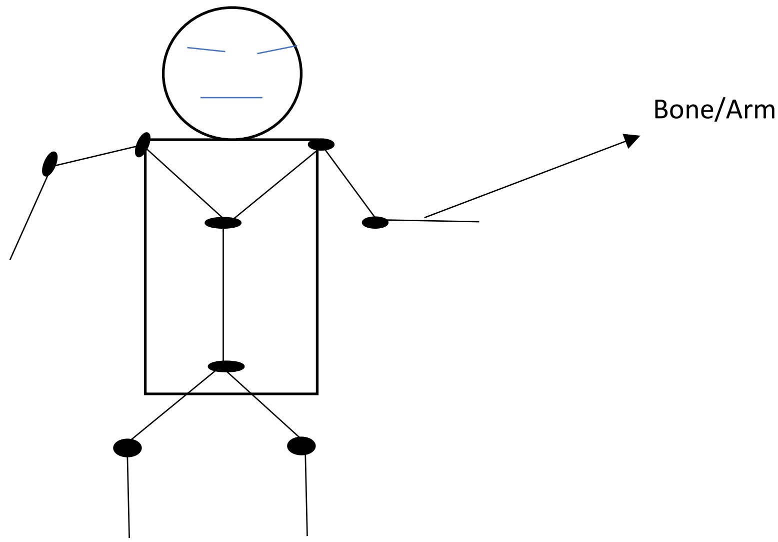
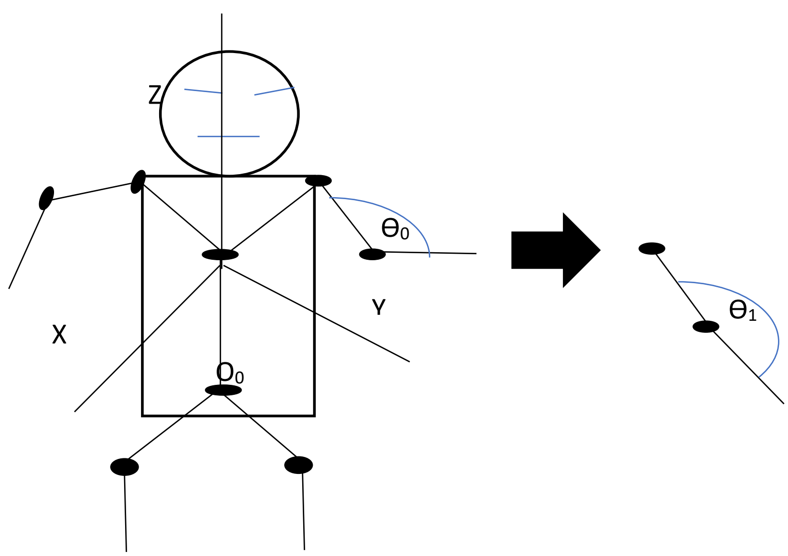
Introduction to Computer Graphics
Lecture Notes
3/13/2019

Homework 2 Part 1: Creating a 3D Human Body

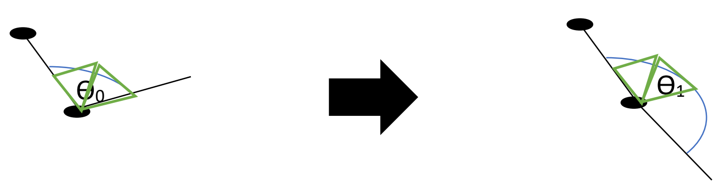


1. To obtain the 3D model, you are allowed to create one or borrow one from online.

2. Remember that we have to use our 3D coordinate system.



The Dilemma with Rotating an Arm

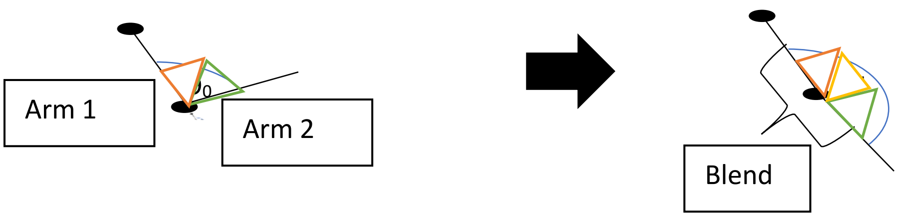


Rotation does not account for the stretch and shrink of objects located on the skeleton. This can cause breakage to occur when rotating human body parts.

Solution

Skinning – LBS – Linear Blend Skinning - https://www.pixelfondue.com/blog/2017/10/27/how-it-works-linear-blend-skinning

For every rotation that we do, we will linearly blend from joint to joint. That is, for every object attached to joint 1 and join 2, upon rotation, will combine features of joint 1 and joint 2 such that there are no gaps of texture between the points.

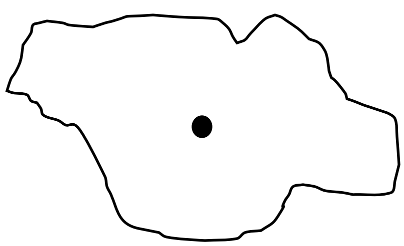


The Steps Required to Complete this Assignment

1. Use OpenGL to display 3D Model.
2. Transform the Skeleton.
3. Apply transformations to the mesh.
4. Apply reflections on simultaneous impact.

Rigid Body Dynamics

Rigid Body – An object that cannot compress or expand.



1. Need three variables for opposition of center of mass and 3 variables for velocity; 6 degrees of freedom and orientation.

State of the Rigid Body

1. Position
2. Velocity
3. Orientation

Cartesian Coordinates

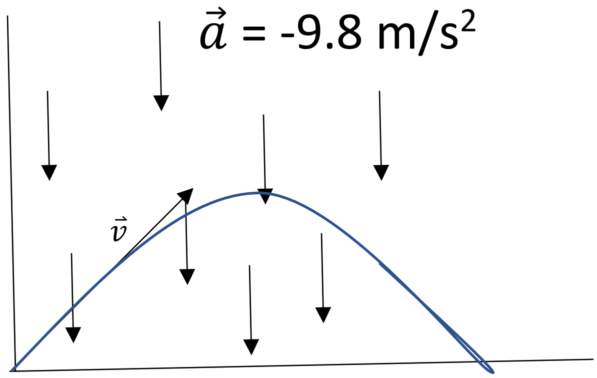
Generalized position talks about location and orientation -> x,y,z coordinates and 4 variables for the unit quaternion.

Linear Motion

1. **Newton's Second Law**

$$\vec{F} = m\vec{a} = \mathrm{m}(\mathrm{d}\vec{v}/\mathrm{dt})$$

Recall the behavior of a projectile in motion on Earth.



Parabolic motion of a projectile.

2. We need to look at two quantities.
   a. $\vec{v} = d\vec{x}/dt$
   b. $\mathrm{m}(\mathrm{d}\vec{v}/\mathrm{dt}) = \vec{F}$ -> Recall the equation for Linear Momentum: $\vec{P} = \mathrm{m}\vec{v}$
   This can be described as

$$\mathrm{d}\vec{P}/\mathrm{dt} = \mathrm{d/dt}(\mathrm{m}\vec{v}) = m\vec{a} = \vec{F}$$

3. Assume that we have: $\vec{x}_0, \vec{v}_0$
   a. $\mathrm{d}\vec{P}/\mathrm{dt} = \vec{F}$, so we can apply **Forward Differencing**

$$= (\vec{P}_1 - \vec{P}_0) / \triangle t = \vec{F}_0 \rightarrow \vec{P}_1 = \vec{P}_0 + \triangle t \; \vec{F}_0$$

$$\vec{P}_1 = \mathrm{m}\vec{v}_1 \rightarrow \vec{v}_1 = \vec{P}_1 / \mathrm{m}$$

   b. **Backward Differencing**

$$d\vec{x}/dt = (\vec{x}_1 - \vec{x}_0) / \triangle t = \vec{v}_1$$

$$\Rightarrow \quad \vec{x}_1 = \vec{x}_0 * \vec{v}_1$$

4. **Forward Differencing vs. Backward Differencing**
   a. Forward differencing requires small time steps.
   b. Backward differencing uses longer time steps.
   c. By using both, we can mitigate the problem of forward differencing exploding with large time steps.

d. Backward differencing mitigates this problem and it is sufficient for simple problems.

Another way to Look at the Linear Momentum Equation

$\vec{v}$ is a vector and m is a scalar.

We can think of m is a matrix,

so m$\vec{v}$ =

$$\begin{matrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{matrix} * \begin{matrix} v1 \\ v2 \\ v3 \end{matrix}$$

Using this, we can define:

5. **Angular Momentum**

$$\vec{L} = \mathrm{I}\vec{w}$$

a. I is the inertia tensor.
b. $\vec{w}$ is the angular velocity.
c. A tensor is just a 3x3 matrix in R^3
d. A scalar quantity is a 0 tensor, vector quantity is a 1 tensor, and a matrix quantity is a 2 tensor.

How to Compute Inertia Tensor of any Object

We want a tensor that is easily inverted.

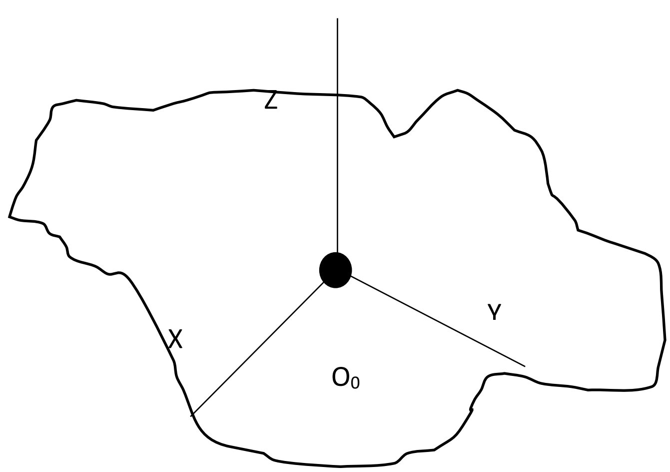
We want to multiply I^{-1} for a new velocity with world space and material space.

To get to the world space, you must be specific with your inertia tensor.

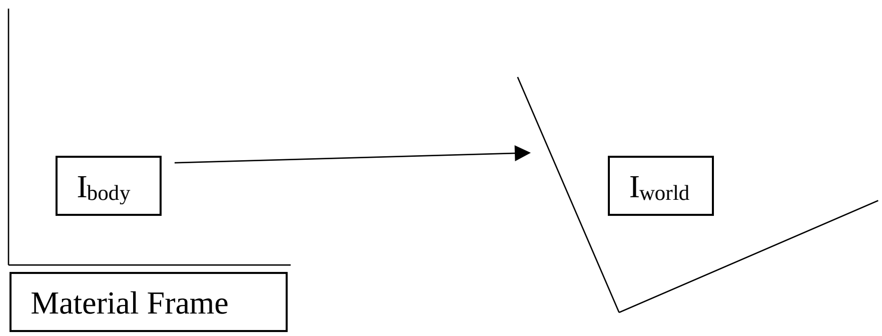
List of Inertial Tensors - https://en.wikipedia.org/wiki/List_of_moments_of_inertia

$$\mathrm{I} = \int_{\Omega 0}^{\Omega n} m\, \vec{r}^2 \mathrm{d}\, \vec{r} = \int_{\Omega 0}^{\Omega n} m \left|\left|\vec{r}\right|\right|_2^2 \mathrm{d}\vec{r}$$

Ω is the area of the object



Inertia Tensor in Material/Body Frame



$$I_{world}(t) = R^T(t)I_{body}R(t)$$

Recall that: $RR^T = I = R^TR$, $R^{-1} = R^T$

- ➔ $I^{-1}_{world}(t) = (R^T(t)I_{body}R(t))^{-1}$
  $= R^T(t)I^{-1}_{body}R(t)$

Torque

$\vec{L} = I\vec{w}$

- ➔ $d\vec{L} / dt = (\vec{L}_1 - \vec{L}_0) / \triangle t = \vec{t}_0 = \text{Torque}$
- ➔ $\vec{L}_1 = \vec{L}_0 + \triangle t\, \vec{t}_0$
- ➔ This is Forward Differencing

Backward Differencing Will be Discussed After Spring Break